Unit 0: Experimental Design and Graph Analysis

Condensed from Modeling Materials @ASU and Experimental Design Graphical Analysis of Data Rice and Schober 2005

Part A: Controlled Experiments

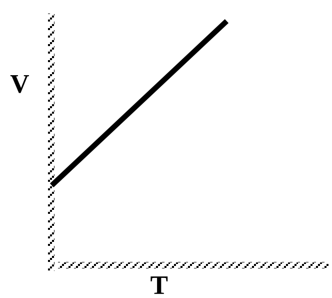
One of the most effective tools for the visual evaluation of data is a graph. The investigator is usually interested in a quantitative graph that shows the relationship between two variables in the form of a "curve" (a curve is just another way of saying "graph", it does not necessarily mean a curved line).

When scientists set up experiments they often attempt to determine how a given variable affects another variable. This requires the experiment to be designed in such a way that when the experimenter changes one variable, the effects of this change on a second variable can be measured. If any other variable that could affect the second variable is changed, the experimenter would have no way of knowing which variable was responsible for the results. For this reason, scientists always attempt to conduct **controlled experiments**. This is done by choosing only one variable to manipulate in an experiment, observing its effect on a second variable, and *holding all other variables in the experiment constant*.

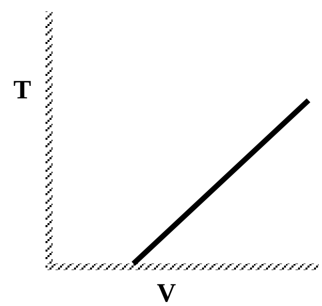
For the relationship y = f(x), **x** is the *independent variable* and **y** is the *dependent variable*. The rectangular coordinate system is convenient for graphing data, with the values of the dependent variable y being plotted along the *vertical axis* and the values of the independent variable x plotted along the *horizontal axis*.

The choice of dependent and independent variables is determined by the experimental approach or the character of the data. Generally, the **independent variable** is the one over which the *experimenter has complete control*; the **dependent variable** is the one that *responds to changes* in the independent variable. An example of this choice might be as follows. In an experiment where a given amount of gas expands when heated at a constant pressure, the relationship between these variables, V and T, may be graphically represented as follows:

By established convention it is proper to plot V = f (T) rather than T = f (V), since the experimenter can directly control the temperature of the gas, but the volume can only be changed by changing the temperature.



Correct



Incorrect

In review, there are only two variables that are allowed to change in a well-designed experiment. The variable manipulated by the experimenter (mass in this example) is called the **independent variable.** The **dependent variable** (period in this case) is the one that responds to or depends on the variable that was manipulated. Any other variable which might affect the value of the dependent value must be held constant. We might call these variables **controlled variables.** When an experiment is conducted with one (and only one) independent variable and one (and only one) dependent variable while holding all other variables constant, it is a **controlled experiment**.

Part B: Graphing Review

Once the data is collected, it is necessary to determine the relationship between the two variables in the experiment. You will construct a graph (or sometimes a series of graphs) from your data in order to determine the relationship between the independent and dependent variables. For each relationship that is being investigated in your experiment, you should prepare the appropriate graph. In general your graphs in physics are of a type known as scatter graphs. The graphs will be used to give you a conceptual understanding of the relation between the variables, and will usually also be used to help you formulate a mathematical statement which describes that relationship. Graphs should include each of the elements described below:

Elements of Good Graphs

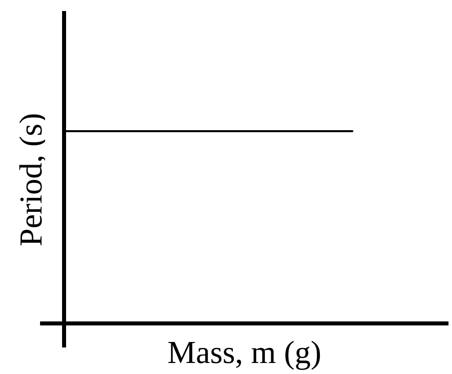
- ✓ A **title** which describes the experiment. This title should be descriptive of the experiment and should indicate the relationship between the variables. It is conventional to title graphs with DEPENDENT VARIABLE vs. INDEPENDENT VARIABLE. For example, if the experiment was designed to show how changing the diameter affects its circumference, the diameter of the circle is the independent variable and the circumference is the dependent variable. An appropriate title is CIRCUMFERENCE vs. DIAMETER FOR CIRCLES.
- ✓ The graph should **fill the space** allotted for the graph. If you have reserved a whole sheet of graph paper for the graph then it should be as large as the paper and proper scaling techniques permit.
- ✓ The graph must be properly **scaled.** The scale for each axis of the graph should always begin at zero. The scale chosen on the axis must be uniform and linear. This means that each square on a given axis must represent the same amount. Obviously each axis for a graph will be scaled independently from the other since they are representing different variables. A given axis must, however, be scaled consistently.
- ✓ Each axis should be **labeled** with the **quantity** being measured and the **units** of measurement. Generally, the independent variable is plotted on the horizontal (or x) axis and the dependent variable is plotted on the vertical (or y) axis.
- ✓ A **line of best fit**. This line should show the overall tendency (or trend) of your data. If the trend is linear, you should draw a straight line which shows that trend using a straight edge. If the trend is a curve, you should sketch a curve which is your best guess as to the tendency of the data. This line (whether straight or curved) does not have to go through all of the data points and it may, in some cases, not go through any of them.
- ✓ Do not, under any circumstances, connect successive data points with a series of straight lines, dot to dot. This makes it difficult to see the overall trend of the data that you are trying to represent.
- ✓ You will choose two points for all linear graphs from which to calculate the **slope** of the line of best fit. (You'll find in physics that the slope of a linear graph is usually some meaningful piece of information related to the experiment.) These points should not be data points unless a data point happens to fall perfectly on the line of best fit. Pick two points which are directly on your line of best fit and which are easy to read from the graph. Be sure to identify these points on the graph.
- ✓ If your graph is not linear (a straight line) you will be expected to manipulate one (or more) of the axes of your graph, re-plot the data and continue doing this until a linear graph is produced.
- ✓ The **E**quation **O**f the **L**ine (EOL) for the **linear** graph must be written using the correct symbols and units.

Part C: Interpreting the Graph and Linearizing

In this course, most of the graphs we make will represent one of four basic relationships between the variables. These are 1) no relation 2) linear relations and 3) square relations and 4) inverse relations. To more specifically describe the relationship between the variables in an experiment, you will be expected to develop an equation. An equation which describes the behavior of a physical system (or any other system for that matter) can be called a **mathematical model.** The information which follows will describe each of the basic types of relationships we tend to see in physics. It also explains the process for **linearizing** non-linear graphs so that the equation of the lone can be written.

1. No Relation

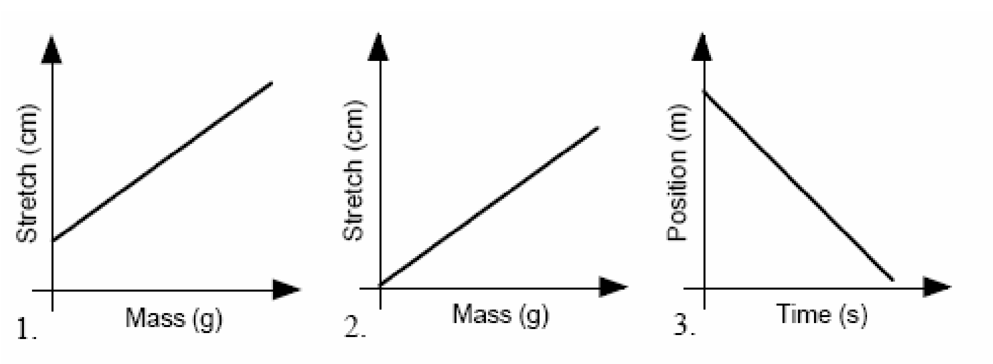
One possible outcome of an experiment is that changing the independent variable will have no effect on the dependent variable. When this happens we say that there is no relationship between the variables. As the independent variable increases, the dependent variable stays the same and the resulting graph is a horizontal line. The slope of a horizontal line is always zero. To the right is a sketch of a graph which shows no relationship between the period of a pendulum and its mass.



2. Linear Relations

Another possible outcome of an experiment is that in which the graphs forms a straight line with a nonzero slope. We call this type of relationship a linear relation. In a linear relation, equal changes in the independent variable result in corresponding constant changes of the dependent variable.

Although all three example graphs are linear, note the differences between the graphs. Graphs 1 and 2 have positive slopes while graph 3 has a negative slope. Graph 2 passes through the origin while graphs 1 and 3 do not. Mathematically, the point where the graph crosses the vertical axis is called the **y-intercept**. In the physical world, the y-intercept has some physical meaning. Specifically, it is the value of the dependent variable when the independent variable is zero. While all three graphs are linear relationships, only one of them illustrates a proportional relationship. A **direct proportion** occurs when, as one variable increases by a certain factor, the other variable increases by the same factor. Graphically, therefore, a direct proportion must not only be linear but must also go through the origin of the axes. When one



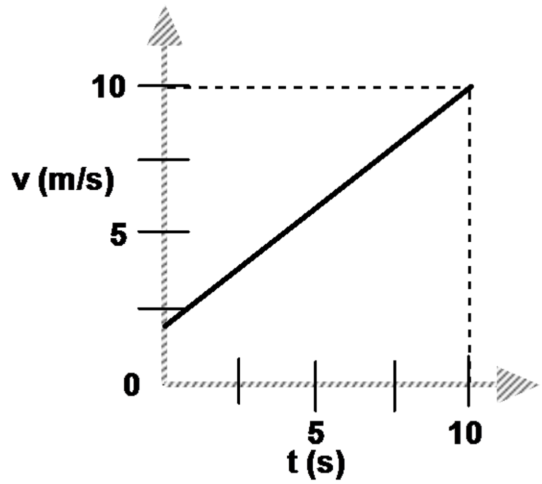
variable is zero, the other variable must also be zero. When one variable doubles, the other variable doubles. When one variable triples, the other variable triples, and so on. Graph 2 is therefore an example of a direct proportion while graphs 1 and 3 are simply linear relationships.

The line on the graph could be represented algebraically by the slope-intercept form: $y = mx + b$,

where:

y is the variable that is plotted on the y axis (usually the dependent variable)
x is the variable that is plotted on the x axis (usually the independent variable)
m is the slope and
b is y-intercept (the value of y when x is equal to zero).

Consider the following graph of velocity vs. time:



The curve is a straight line, indicating a linear relationship between the two variables. Therefore,

$$\mathbf{v} = m\mathbf{t} + b,$$

where slope = $m = \dfrac{\Delta v}{\Delta t} = \dfrac{v_2 - v_1}{t_2 - t_1}$

From the graph,

$$m = \frac{8.0 \text{ m/s}}{10.0 \text{ s}} = 0.80 \text{ m/s/s}.$$

The curve intercepts the v-axis at v = 2.0 m/s. This indicates that the velocity was 2.0 m/s when the first measurement was taken; that is, when t = 0. Thus, $b = v_o = 2.0$ m/s.

The **general or generic** equation, **v** = m**t** + b, can then be rewritten as an equation that is **specific** for this graph, shown below.

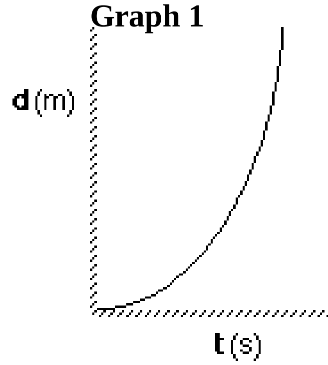
$\mathbf{v} = (0.80 \text{ m/s/s})\mathbf{t} + 2.0 \text{ m/s}.$ (This is the EOL.)

Notice that the **generic** equation **does not include** any numbers or units. However, when you look at the **specific** equation it **does include** numbers and the associated units.

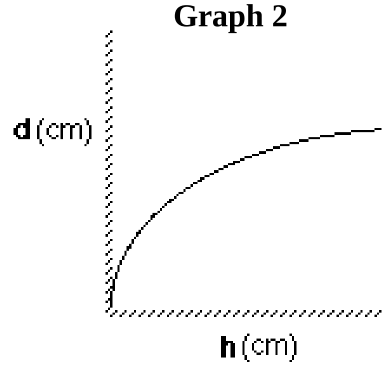
3. Non-linear Graphs

Graphs 1 and 2 are examples of non-linear relationships. While many mathematical relationships could yield graphs like these, we will find that in most of our experiments in this course, these graphs are basic ***power*** curves. One way to think of this curve is that one axis is "more powerful" and bends the curve towards itself. This also leads us to how to **linearize** the graph.

Graph 1



Graph 2



Graph 1 (on page 4) appears to be a top-opening parabola. This means the vertical axis appears to be “more powerful”. In order to linearize this graph, a second plot (a test plot) is completed in which all of the horizontal data has been raised to the second power. The second graph made is d vs. t^2. The resulting graph is shown below:

Since the plot of d vs. t^2 is linear, $\mathbf{d} = m\mathbf{t}^2 + b$.

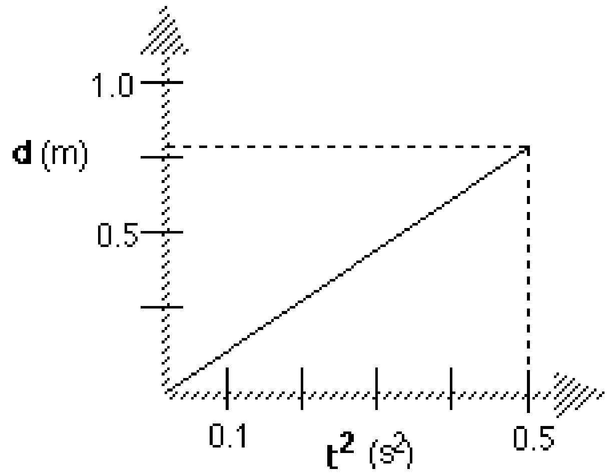
The slope, m, is calculated by

$$m = \frac{\Delta d}{\Delta t^2} = \frac{.80\text{m}}{.50\text{s}^2} = 1.6\ \text{m/s}^2$$

Since the curve passes through the origin, b = 0.

The model that describes the relationship is

$$\mathbf{d} = (1.6\ \text{m/s}^2)\mathbf{t}^2\ .$$



In Graph 2 (on page 4) the horizontal axis appears to be “more powerful” resulting in a side opening parabola. In order to linearize, a second plot is completed in which all the vertical data has been raised to the second power. Since the graph of d^2vs. h is linear the **generic** equation is

$$\mathbf{d}^2 = m\mathbf{h} + b.$$

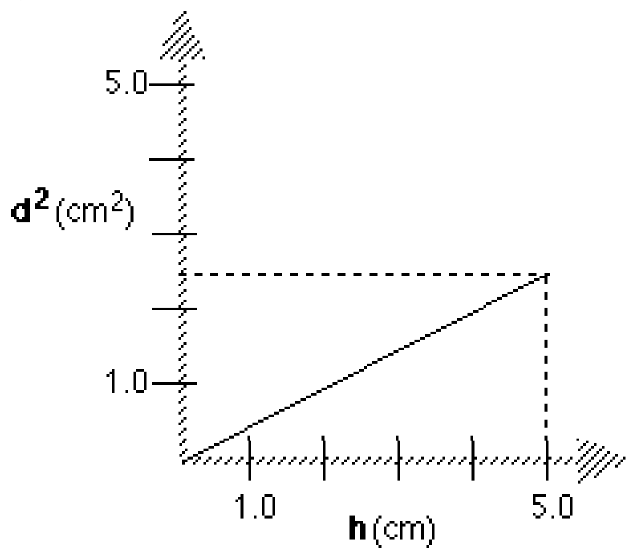
The slope, m, is

$$m = \frac{\Delta d^2}{\Delta h} = \frac{2.5\text{cm}^2}{5.0\text{ cm}} = 0.50\text{ cm}.$$

Since the curve passes through the origin, b = 0.

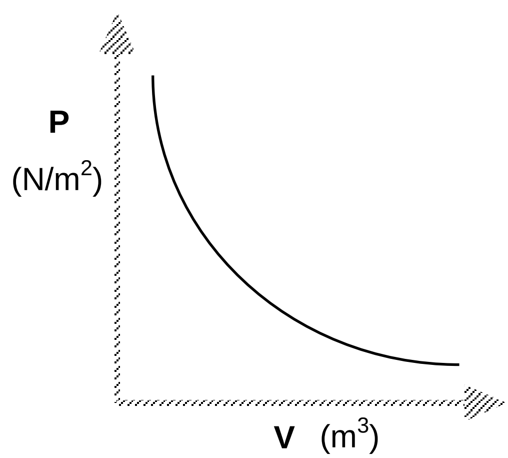
The **specific** equation is then:

$$\mathbf{d}^2 = (0.50\text{ cm})\mathbf{h}.$$

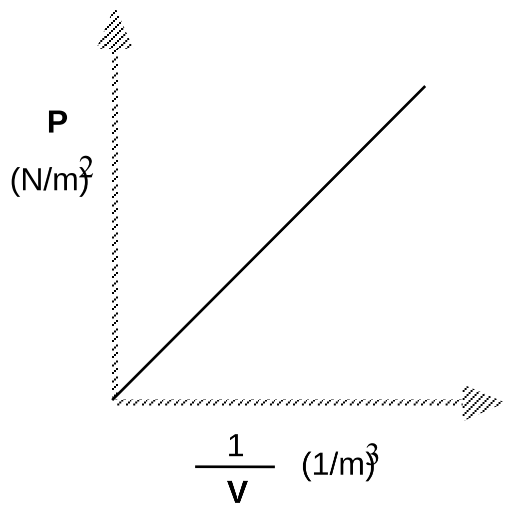


4. Inverse Relations

The final type of fundamental relationship that we will study is the inverse relation. In an inverse relationship, as the independent variable increases the dependent variable decreases. This can take multiple forms, but the most common type that you will encounter in this physics course looks like the graph to the right.



As volume of a trapped gas increases, the pressure of the gas decreases. What mathematical manipulation of the data might possibly linearize the graph to allow us do develop a mathematical model? The answer might be to take the reciprocal of one of the variables. This will cause the variable you have manipulated to decrease if it was increasing or to increase if it was decreasing. A second graph of P vs. $\frac{1}{V}$ should be made to **linearize** (make it into a linear graph) the graph. The resulting graph is shown below:



The equation for this straight line is:

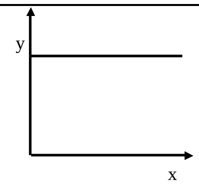
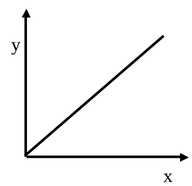
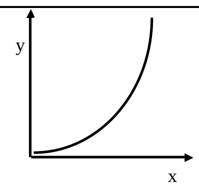
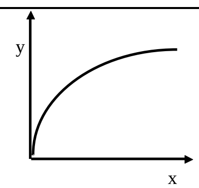
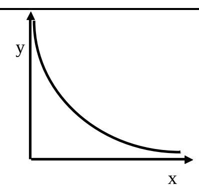
$$P = m\left(\frac{1}{V}\right) + b,$$

where b = 0. Therefore; $P = \frac{m}{V}$; when rearranged, this yields PV = constant, which is known as Boyle's Law.

In conclusion, a graph is one of the most effective representations of the relationship between two variables. The **independent variable** (one controlled by the experimenter) is usually placed on the x-axis. The **dependent variable** (one that responds to changes in the independent variable) is usually placed on the y-axis. You will be expected to learn to describe the relationship between the variables on a graph in two ways. One way will be to give a written statement of the general relationship between the two variables. The second is to develop an equation which will describe the relationship between these variables mathematically. We will call this equation a mathematical model of the physical relationship.

Graphical Methods-Summary

Adding a Calculated Column (Modifying a Data Set—Linearizing):

Graph shape	Written relationship	Modification required to linearize graph	Algebraic representation
	***No Relationship*** As x increases, y remains the same. There is no relationship between the variables.	None	$y = b$, or y is constant
	***Linear relationship*** As x increases, y increases proportionally. Y is directly proportional to x.	None	$y = mx + b$
	***Square Relationship*** Y is proportional to the square of x.	Raise the horizontal data to the second power. Graph y vs x^2	$y = mx^2 + b$
	***Square Relationship*** The square of y is proportional to x.	Raise the vertical data to the second power. Graph y^2 vs x	$y^2 = mx + b$
	***Inverse Relationship*** As x increases, y decreases. Y is inversely proportional to x.	Take the inverse of the horizontal data or raise horizontal data to -1 power. Graph y vs $\frac{1}{x}$,	$y = m\left(\frac{1}{x}\right) + b$

1. Under the **Data** menu, select **New Calculated Column**.
2. Simply use the Dialog Box to modify the column.
3. Notice in this example the modification is to square the Time. The new label, new symbol and new units have all been entered.
4. For the Equation, the Time column was selected from the Variables drop-down.
5. The new calculated column will be automatically plotted on the original graph.